MOLIEN SERIES RELATED TO CERTAIN FINITE UNITARY REFLECTION GROUPS

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§1 Introduction.

In their joint work[1], Bannai and Ozeki considered the invariant ring related to certain finite reflection groups.

Let $G =< \sigma_1, \sigma_2 >$ be the subgroup of $GL(4, \mathbb{C})$ generated by σ_1, σ_2, where

$$\sigma_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 & & \\ 1 & -1 & & \\ & & 1 & 1 \\ & & 1 & -1 \end{pmatrix} \text{ and } \sigma_2 = \begin{pmatrix} \sqrt{-1} & & & \\ & 1 & & \\ & & \sqrt{-1} & \\ & & & 1 \end{pmatrix}.$$

Then $|G| = 192$ and G is isomorphic to the finite unitary group No. 9 generated by reflections (u.g.g.r.) in Shephard-Todd [14]. Let $R = \mathbb{C}[x_1, x_2, y_1, y_2]$ and let R^G be the ring of invariants under the group $G =< \sigma_1, \sigma_2 >$ with the natural action. They calculated the Molien series for R^G, as well as the Molien series for finite u.g.g.r. No. 8 in [14].

Let G be one of the reflection groups in the Shephard-Todd list and let

$$\tilde{G} = \{\begin{pmatrix} g & \\ & g \end{pmatrix} |g \in G\}.$$

The purpose of this paper is to give the Molien series $\Phi_{\tilde{G}}(\lambda) = \sum_{n\geq 0}(\dim R_n^{\tilde{G}})\lambda^n$ when G is one of the groups No.4∼No.37 in [14]. (For No.1∼No.3, see Tanabe[18].) We frequently use **GAP** [11] throughout this paper.

§2 Invariant rings of finite groups and reflection groups.

In this sectin, we state some well-known facts without proofs. (see [17].)

Let G be an $l \times l$ matrix group of finite order. G acts on $R = \mathbb{C}[x_1, \ldots, x_l]$ in a natural way. Set

$$R^G = \{f \in R|\sigma \circ f = f, \forall \sigma \in G\}$$

and

$$\Phi_G(\lambda) = \sum_{n\geq 0}(\dim R_n^G)\lambda^n,$$

where R_n^G denotes the homogeneous part of degree n in R^G.

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Theorem 2.1 (Molien, see [15], [16], [17]).

$$\Phi_G(\lambda) = \frac{1}{|G|} \sum_{\sigma \in G} \frac{1}{\det(I - \lambda\sigma)},$$

where I denotes an identity matrix.

R^G is a Cohen-Macaulay ring and we have the Hironaka decomposition (see [17])

$$R^G = \oplus_{i=1}^{s} \eta_i \mathbb{C}[\theta_1, \ldots, \theta_l], \text{ for some } s \in \mathbb{Z}_{\geq 1}$$

where $\eta_1 = 1$, and $\theta_1, \ldots, \theta_l$ are called a homogeneous system of parameters (h.s.o.p.) of G. From the expression of the Hironaka decomposion, the Molien series is

$$\Phi_G(\lambda) = \frac{\sum_{i=1}^{s} \lambda^{\deg(\eta_i)}}{(1 - \lambda^{\deg(\theta_1)}) \ldots (1 - \lambda^{\deg(\theta_l)})}.$$

Let V be a unitary space of dimension l. A *reflection* is a linear transformation of finite order in V whose eigenvalues are all equal to unity except one primitive m^{th} root of unity, ζ_m. A *reflection group* is a group generated by reflections.

The next theorem is one of the most fundamental results in this area.

Theorem 2.2 (Chevalley, Shephard-Todd [3], [4], [14]). *The invariant ring $R^G = \mathbb{C}[x_1, \ldots, x_l]^G$ of a finite matrix group $G \subset GL(V)$ is generated by n algebraically independent homogeneous invariants if and only if G is a reflection group.*

§3 **List of results.**

We need some preparation.

Lemma 3.1. *Let $\theta_1(x_1, \ldots, x_l), \ldots, \theta_l(x_1, \ldots, x_l)$ be a h.s.o.p. of G. Then $\theta_1(x_1, \ldots, x_l), \ldots,$ $\theta_l(x_1, \ldots, x_l)$, $\theta_1(y_1, \ldots, y_l), \ldots, \theta_l(y_1, \ldots, y_l)$ are a h.s.o.p. of $\tilde{G}$.*

For a proof, see Tanabe[18].

From this lemma, we can find the denominator of $\Phi_{\tilde{G}}(\lambda)$.

We can also obtain information on conjugacy classes of G, which makes our computation easier, from [7], [8], [9], [10], and [19].

Here we give two ways of calculating the Molien series.

First, we consider the group of $No.9$. (As a matter of fact, I calculated the Molien series in this way for all cases except for $No.34$.) By Lemma 3.1 and the fact that the degrees of θ_i of this group is 8 and 24, we know that $\Phi_{\tilde{G}}(\lambda)$ can be written in the form $\frac{\star\star\star\star\star\star\star\star}{(1-\lambda^8)^2(1-\lambda^{24})^2}$. Let $\{C_i\}_{1 \leq i \leq 32}$ and $\{c_i\}_{1 \leq i \leq 32}$ be conjugacy classes of G (not $\tilde{G}$!) and their representatives, respectively. (See the appendix. The numbering has no meaning.) By Theorem 2.1,

$$
\begin{aligned}
\Phi_{\tilde{G}}(\lambda)(1-\lambda^8)^2(1-\lambda^{24})^2 &= \frac{(1-\lambda^8)^2(1-\lambda^{24})^2}{192}\sum_{1\le i\le 32}\frac{|C_i|}{\det(I-\lambda c_i)^2} \\
&= \frac{(1-\lambda^8)^2(1-\lambda^{24})^2}{192}[\\
&\frac{1}{(\lambda^2-2\lambda+1)^2}\frac{1}{(\lambda^2+2\lambda+1)^2}+\frac{6}{(-\lambda^2+1)^2}+\frac{12}{(-\lambda^2+1)^2}+\frac{8}{(\lambda^2+\lambda+1)^2} \\
&+\frac{1}{(-\lambda^2-2\zeta_4\lambda+1)^2}+\frac{1}{(-\lambda^2+2\zeta_4\lambda+1)^2}+\frac{6}{(\lambda^2+1)^2}+\frac{6}{(\zeta_4\lambda^2+(1+\zeta_4)\lambda+1)^2} \\
&+\frac{6}{(\zeta_4\lambda^2-(1+\zeta_4)\lambda+1)^2}+\frac{6}{(-\zeta_4\lambda^2+(-1+\zeta_4)\lambda+1)^2}+\frac{6}{(-\zeta_4\lambda^2+(1-\zeta_4)\lambda+1)^2} \\
&+\frac{12}{(\lambda^2+1)^2}+\frac{8}{(\lambda^2-\lambda+1)^2}+\frac{1}{(\zeta_4\lambda^2-2\zeta_8\lambda+1)^2}+\frac{1}{(\zeta_4\lambda^2+2\zeta_8\lambda+1)^2} \\
&+\frac{1}{(-\zeta_4\lambda^2-2\zeta_8^3\lambda+1)^2}+\frac{1}{(-\zeta_4\lambda^2+2\zeta_8^3\lambda+1)^2}+\frac{6}{(\zeta_4\lambda^2+1)^2}+\frac{6}{(-\zeta_4\lambda^2+1)^2}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{6}{(\lambda^2+(-\zeta_8+\zeta_8^3)\lambda+1)^2}+\frac{6}{(\lambda^2+(\zeta_8-\zeta_8^3)\lambda+1)^2}+\frac{6}{(-\lambda^2+(\zeta_8+\zeta_8^3)\lambda+1)^2} \\
&+\frac{6}{(-\lambda^2-(\zeta_8+\zeta_8^3)\lambda+1)^2}+\frac{12}{(-\zeta_4\lambda^2+1)^2}+\frac{12}{(\zeta_4\lambda^2+1)^2} \\
&+\frac{8}{(-\lambda^2+\zeta_4\lambda+1)^2}+\frac{8}{(-\lambda^2-\zeta_4\lambda+1)^2}+\frac{8}{(\zeta_4\lambda^2-\zeta_8\lambda+1)^2} \\
&+\frac{8}{(-\zeta_4\lambda^2-\zeta_8^3\lambda+1)^2}+\frac{8}{(\zeta_4\lambda^2+\zeta_8\lambda+1)^2}+\frac{8}{(-\zeta_4\lambda^2+\zeta_8^3\lambda+1)^2}] \\
&= 1+8\lambda^8+21\lambda^{16}+58\lambda^{24}+47\lambda^{32}+35\lambda^{40}+21\lambda^{48}+\lambda^{56}
\end{aligned}
$$

i.e.,

$$
(\star\star)\qquad \Phi_{\tilde{G}}(\lambda)=\frac{1+8\lambda^8+21\lambda^{16}+58\lambda^{24}+47\lambda^{32}+35\lambda^{40}+21\lambda^{48}+\lambda^{56}}{(1-\lambda^8)^2(1-\lambda^{24})^2}.
$$

This gives information on the Hironaka decomposition as follows. As is well known, the basic invariants of G (not $\tilde{G}$!) are $x^8+14x^4y^4+y^8$ and $x^4y^4(x^4-y^4)^4$: $\mathbb{C}[x,\,y]^G=\mathbb{C}[x^8+14x^4y^4+y^8, x^4y^4(x^4-y^4)^4]$ (*cf.* [15]). By Lemma 3.1, we may set $\theta_1=x_1^8+14x_1^4y_1^4+y_1^8, \theta_2=x_1^8+14x_2^4y_2^4+y_2^8, \theta_3=x_1^4y_1^4(x_1^4-y_1^4)^4, \theta_4=x_2^4y_2^4(x_2^4-y_2^4)^4$. According to the form of $\Phi_{\tilde{G}}(\lambda)(1-\lambda^8)^2(1-\lambda^{24})^2$, we may set, for example, $\eta_1=1, \deg(\eta_i)=8(2\le i\le 9), \deg(\eta_i)=16(10\le i\le 30), \deg(\eta_i)=24(31\le i\le 88), \deg(\eta_i)=32(89\le i\le 135), \deg(\eta_i)=40(136\le i\le 170), \deg(\eta_i)=48(171\le i\le 191)$, and $\deg(\eta_{192})=56$. (Although we know the degrees of η_i's, the author does not know the explicit form of η_i's at the time of writing.)
In the list below, we denote by $\Phi_i(\lambda)\,(4\le i\le 37)$ the Molien series of $R^{\tilde{G}}$ where G is the u.g.g.r. $No.i$ in [14], and we give the numerator of $\Phi_{\tilde{G}}(\lambda)$, that is, $\Phi_{\tilde{G}}(\lambda)\Pi_{i=1}^{l}(1-\lambda^{\deg(\theta_i)})^2$.

Second, we consider the group of $No.34$.

Let G be the u.g.g.r. $No.34$ in [14], of order $108\cdot 9!$. G is considered as the matrix group of degree 6 (*cf.* [12], [14]) and its center Z is a cyclic group of order 6, consisting of scalar matrices. Hamill [10] determined the conjugacy classes of the group G/Z and we calculate the Molien series for $\tilde{G}$, using his or her result (*cf.* [19]). Let $\{D_\alpha\}_\alpha$ and $\{d_\alpha Z\}_\alpha$ be conjugacy classes of G/Z and their representatives, respectively. Then,

$$
\Phi_{\tilde{G}}(\lambda)=\frac{1}{|G|}\sum_{\alpha}|D_\alpha|\sum_{1\le i\le 6}\frac{1}{\phi(\zeta_6^i d_\alpha)^2},
$$

where $\phi(d) = \det(I - \lambda d)$. We give D_α's and $\phi(d_\alpha)$'s explicitly at the appendix (The numbering follows [10], [19].). As the degrees of θ_i of this group are 6, 12, 18, 24, 30, and 42, we calculate $\Phi_{\tilde{G}}(\lambda)$ by Theorem 2.1 and we have

$$\Phi_{34}(\lambda)(1-\lambda^6)^2(1-\lambda^{12})^2(1-\lambda^{18})^2(1-\lambda^{24})^2(1-\lambda^{30})^2(1-\lambda^{42})^2 = 1+5\lambda^6+26\lambda^{12}+121\lambda^{18}+493\lambda^{24}+1708\lambda^{30}+5171\lambda^{36}+13870\lambda^{42}+33231\lambda^{48}+72049\lambda^{54}+142690\lambda^{60}+260289\lambda^{66}+440521\lambda^{72}+695882\lambda^{78}+1031256\lambda^{84}+1439572\lambda^{90}+1899382\lambda^{96}+2374985\lambda^{102}+2820136\lambda^{108}+3185028\lambda^{114}+3424853\lambda^{120}+3508502\lambda^{126}+3424853\lambda^{132}+3185028\lambda^{138}+2820136\lambda^{144}+2374985\lambda^{150}+1899382\lambda^{156}+1439572\lambda^{162}+1031256\lambda^{168}+695882\lambda^{174}+440521\lambda^{180}+260289\lambda^{186}+142690\lambda^{192}+72049\lambda^{198}+33231\lambda^{204}+13870\lambda^{210}+5171\lambda^{216}+1708\lambda^{222}+493\lambda^{228}+121\lambda^{234}+26\lambda^{240}+5\lambda^{246}+\lambda^{252}.$$

As in the $No.9$ case, we can read off $\deg(\eta_j)$'s from this formula.

List. $\Phi_4(\lambda)(1-\lambda^4)^2(1-\lambda^6)^2 = 1+3\lambda^4+6\lambda^6+5\lambda^8+3\lambda^{10}+5\lambda^{12}+\lambda^{14}$

$\Phi_5(\lambda)(1-\lambda^6)^2(1-\lambda^{12})^2 = 1+6\lambda^6+26\lambda^{12}+23\lambda^{18}+15\lambda^{24}+\lambda^{30}$

$\Phi_6(\lambda)(1-\lambda^4)^2(1-\lambda^{12})^2 = 1+3\lambda^4+5\lambda^8+18\lambda^{12}+9\lambda^{16}+7\lambda^{20}+5\lambda^{24}$

$\Phi_7(\lambda)(1-\lambda^{12})^4 = 1+39\lambda^{12}+87\lambda^{24}+17\lambda^{36}$

$\Phi_8(\lambda)(1-\lambda^8)^2(1-\lambda^{12})^2 = 1+8\lambda^8+18\lambda^{12}+21\lambda^{16}+19\lambda^{20}+21\lambda^{24}+7\lambda^{28}+\lambda^{32}$

$\Phi_9(\lambda)(1-\lambda^8)^2(1-\lambda^{24})^2 = 1+8\lambda^8+21\lambda^{16}+58\lambda^{24}+47\lambda^{32}+35\lambda^{40}+21\lambda^{48}+\lambda^{56}$

$\Phi_{10}(\lambda)(1-\lambda^{12})^2(1-\lambda^{24})^2 = 1+18\lambda^{12}+89\lambda^{24}+107\lambda^{36}+66\lambda^{48}+7\lambda^{60}$

$\Phi_{11}(\lambda)(1-\lambda^{24})^4 = 1+126\lambda^{24}+369\lambda^{48}+80\lambda^{72}$

$\Phi_{12}(\lambda)(1-\lambda^6)^2(1-\lambda^8)^2 = 1+\lambda^4+5\lambda^6+8\lambda^8+5\lambda^{10}+8\lambda^{12}+5\lambda^{14}+8\lambda^{16}+5\lambda^{18}+\lambda^{20}+\lambda^{24}$

$\Phi_{13}(\lambda)(1-\lambda^8)^2(1-\lambda^{12})^2 = 1+\lambda^4+8\lambda^8+19\lambda^{12}+19\lambda^{16}+19\lambda^{20}+19\lambda^{24}+8\lambda^{28}+\lambda^{32}+\lambda^{36}$

$\Phi_{14}(\lambda)(1-\lambda^6)^2(1-\lambda^{24})^2 = 1+5\lambda^6+10\lambda^{12}+15\lambda^{18}+43\lambda^{24}+25\lambda^{30}+20\lambda^{36}+15\lambda^{42}+10\lambda^{48}$

$\Phi_{15}(\lambda)(1-\lambda^{12})^2(1-\lambda^{24})^2 = 1+21\lambda^{12}+83\lambda^{24}+113\lambda^{36}+60\lambda^{48}+10\lambda^{60}$

$\Phi_{16}(\lambda)(1-\lambda^{20})^2(1-\lambda^{30})^2 = 1+\lambda^{10}+33\lambda^{20}+87\lambda^{30}+131\lambda^{40}+143\lambda^{50}+131\lambda^{60}+58\lambda^{70}+14\lambda^{80}+\lambda^{90}$

$\Phi_{17}(\lambda)(1-\lambda^{20})^2(1-\lambda^{60})^2 = 1+33\lambda^{20}+133\lambda^{40}+306\lambda^{60}+333\lambda^{80}+247\lambda^{100}+133\lambda^{120}+14\lambda^{140}$

$\Phi_{18}(\lambda)(1-\lambda^{30})^2(1-\lambda^{60})^2 = 1+89\lambda^{30}+494\lambda^{60}+721\lambda^{90}+435\lambda^{120}+60\lambda^{150}$

$\Phi_{19}(\lambda)(1-\lambda^{60})^4 = 1+673\lambda^{60}+2371\lambda^{120}+555\lambda^{180}$

$\Phi_{20}(\lambda)(1-\lambda^{12})^2(1-\lambda^{30})^2 = 1+\lambda^6+12\lambda^{12}+12\lambda^{18}+33\lambda^{24}+62\lambda^{30}+53\lambda^{36}+53\lambda^{42}+53\lambda^{48}+33\lambda^{54}+33\lambda^{60}+12\lambda^{66}+\lambda^{72}+\lambda^{78}$

$\Phi_{21}(\lambda)(1-\lambda^{12})^2(1-\lambda^{60})^2 = 1+12\lambda^{12}+33\lambda^{24}+55\lambda^{36}+77\lambda^{48}+158\lambda^{60}+119\lambda^{72}+99\lambda^{84}+77\lambda^{96}+55\lambda^{108}+33\lambda^{120}+\lambda^{132}$

$\Phi_{22}(\lambda)(1-\lambda^{12})^2(1-\lambda^{20})^2 = 1+\lambda^4+\lambda^8+12\lambda^{12}+12\lambda^{16}+31\lambda^{20}+31\lambda^{24}+31\lambda^{28}+31\lambda^{32}+31\lambda^{36}+31\lambda^{40}+12\lambda^{44}+12\lambda^{48}+\lambda^{52}+\lambda^{56}+\lambda^{60}$

$\Phi_{23}(\lambda)(1-\lambda^2)^2(1-\lambda^6)^2(1-\lambda^{10})^2 = 1+\lambda^2+\lambda^4+6\lambda^6+6\lambda^8+15\lambda^{10}+15\lambda^{12}+15\lambda^{14}+15\lambda^{16}+15\lambda^{18}+15\lambda^{20}+6\lambda^{22}+6\lambda^{24}+\lambda^{26}+\lambda^{28}+\lambda^{30}$

$\Phi_{24}(\lambda)(1-\lambda^4)^2(1-\lambda^6)^2(1-\lambda^{14})^2 = 1+3\lambda^4+5\lambda^6+6\lambda^8+10\lambda^{10}+19\lambda^{12}+29\lambda^{14}+27\lambda^{16}+33\lambda^{18}+35\lambda^{20}+35\lambda^{22}+33\lambda^{24}+27\lambda^{26}+29\lambda^{28}+19\lambda^{30}+10\lambda^{32}+6\lambda^{34}+5\lambda^{36}+3\lambda^{38}+\lambda^{42}$

$\Phi_{25}(\lambda)(1-\lambda^6)^2(1-\lambda^9)^2(1-\lambda^{12})^2 = 1+5\lambda^6+12\lambda^9+33\lambda^{12}+42\lambda^{15}+82\lambda^{18}+88\lambda^{21}+117\lambda^{24}+92\lambda^{27}+82\lambda^{30}+44\lambda^{33}+35\lambda^{36}+10\lambda^{39}+5\lambda^{42}$

$\Phi_{26}(\lambda)(1-\lambda^6)^2(1-\lambda^{12})^2(1-\lambda^{18})^2 = 1+5\lambda^6+33\lambda^{12}+107\lambda^{18}+206\lambda^{24}+291\lambda^{30}+301\lambda^{36}+210\lambda^{42}+102\lambda^{48}+35\lambda^{54}+5\lambda^{60}$

$\Phi_{27}(\lambda)(1-\lambda^6)^2(1-\lambda^{12})^2(1-\lambda^{30})^2 = 1+5\lambda^6+26\lambda^{12}+61\lambda^{18}+132\lambda^{24}+227\lambda^{30}+289\lambda^{36}+339\lambda^{42}+339\lambda^{48}+289\lambda^{54}+227\lambda^{60}+132\lambda^{66}+61\lambda^{72}+26\lambda^{78}+5\lambda^{84}+\lambda^{90}$

$\Phi_{28}(\lambda)(1-\lambda^2)^2(1-\lambda^6)^2(1-\lambda^8)^2(1-\lambda^{12})^2 = 1+\lambda^2+\lambda^4+6\lambda^6+14\lambda^8+14\lambda^{10}+42\lambda^{12}+47\lambda^{14}+68\lambda^{16}+92\lambda^{18}+111\lambda^{20}+104\lambda^{22}+150\lambda^{24}+104\lambda^{26}+111\lambda^{28}+92\lambda^{30}+68\lambda^{32}+47\lambda^{34}+42\lambda^{36}+14\lambda^{38}+14\lambda^{40}+6\lambda^{42}+\lambda^{44}+\lambda^{46}+\lambda^{48}$

$\Phi_{29}(\lambda)(1-\lambda^4)^2(1-\lambda^8)^2(1-\lambda^{12})^2(1-\lambda^{20})^2 = 1+3\lambda^4+13\lambda^8+43\lambda^{12}+106\lambda^{16}+236\lambda^{20}+409\lambda^{24}+628\lambda^{28}+848\lambda^{32}+1014\lambda^{36}+1078\lambda^{40}+1014\lambda^{44}+848\lambda^{48}+628\lambda^{52}+409\lambda^{56}+236\lambda^{60}+106\lambda^{64}+43\lambda^{68}+13\lambda^{72}+3\lambda^{76}+\lambda^{80}$

$\Phi_{30}(\lambda)(1-\lambda^2)^2(1-\lambda^{12})^2(1-\lambda^{20})^2(1-\lambda^{30})^2 = 1+\lambda^2+\lambda^4+\lambda^6+\lambda^8+\lambda^{10}+13\lambda^{12}+13\lambda^{14}+13\lambda^{16}+13\lambda^{18}+46\lambda^{20}+46\lambda^{22}+79\lambda^{24}+79\lambda^{26}+79\lambda^{28}+168\lambda^{30}+198\lambda^{32}+198\lambda^{34}+253\lambda^{36}+253\lambda^{38}+386\lambda^{40}+425\lambda^{42}+436\lambda^{44}+436\lambda^{46}+513\lambda^{48}+655\lambda^{50}+598\lambda^{52}+631\lambda^{54}+620\lambda^{56}+620\lambda^{58}+846\lambda^{60}+620\lambda^{62}+620\lambda^{64}+631\lambda^{66}+598\lambda^{68}+655\lambda^{70}+513\lambda^{72}+436\lambda^{74}+436\lambda^{76}+425\lambda^{78}+386\lambda^{80}+253\lambda^{82}+253\lambda^{84}+198\lambda^{86}+198\lambda^{88}+168\lambda^{90}+79\lambda^{92}+79\lambda^{94}+79\lambda^{96}+46\lambda^{98}+46\lambda^{100}+13\lambda^{102}+13\lambda^{104}+13\lambda^{106}+13\lambda^{108}+\lambda^{110}+\lambda^{112}+\lambda^{114}+\lambda^{116}+\lambda^{118}+\lambda^{120}$

$\Phi_{31}(\lambda)(1-\lambda^8)^2(1-\lambda^{12})^2(1-\lambda^{20})^2(1-\lambda^{24})^2 = 1+8\lambda^8+19\lambda^{12}+49\lambda^{16}+136\lambda^{20}+286\lambda^{24}+499\lambda^{28}+918\lambda^{32}+1439\lambda^{36}+2069\lambda^{40}+2822\lambda^{44}+3621\lambda^{48}+4147\lambda^{52}+4616\lambda^{56}+4820\lambda^{60}+4616\lambda^{64}+4147\lambda^{68}+3621\lambda^{72}+2822\lambda^{76}+2069\lambda^{80}+1439\lambda^{84}+918\lambda^{88}+499\lambda^{92}+286\lambda^{96}+136\lambda^{100}+49\lambda^{104}+19\lambda^{108}+8\lambda^{112}+\lambda^{120}$

$\Phi_{32}(\lambda)(1-\lambda^{12})^2(1-\lambda^{18})^2(1-\lambda^{24})^2(1-\lambda^{30})^2 = 1+12\lambda^{12}+47\lambda^{18}+181\lambda^{24}+513\lambda^{30}+1258\lambda^{36}+2617\lambda^{42}+4845\lambda^{48}+7829\lambda^{54}+11438\lambda^{60}+14973\lambda^{66}+17874\lambda^{72}+19301\lambda^{78}+19079\lambda^{84}+17086\lambda^{90}+13951\lambda^{96}+10248\lambda^{102}+6806\lambda^{108}+3986\lambda^{114}+2089\lambda^{120}+916\lambda^{126}+347\lambda^{132}+99\lambda^{138}+23\lambda^{144}+\lambda^{150}$

$\Phi_{33}(\lambda)(1-\lambda^4)^2(1-\lambda^6)^2(1-\lambda^{10})^2(1-\lambda^{12})^2(1-\lambda^{18})^2 = 1+3\lambda^4+5\lambda^6+6\lambda^8+24\lambda^{10}+43\lambda^{12}+57\lambda^{14}+127\lambda^{16}+211\lambda^{18}+275\lambda^{20}+472\lambda^{22}+693\lambda^{24}+842\lambda^{26}+1218\lambda^{28}+1630\lambda^{30}+1826\lambda^{32}+2295\lambda^{34}+2818\lambda^{36}+2919\lambda^{38}+3258\lambda^{40}+3671\lambda^{42}+3526\lambda^{44}+3526\lambda^{46}+3671\lambda^{48}+3258\lambda^{50}+2919\lambda^{52}+2818\lambda^{54}+2295\lambda^{56}+1826\lambda^{58}+1630\lambda^{60}+1218\lambda^{62}+842\lambda^{64}+693\lambda^{66}+472\lambda^{68}+275\lambda^{70}+211\lambda^{72}+127\lambda^{74}+57\lambda^{76}+43\lambda^{78}+24\lambda^{80}+6\lambda^{82}+5\lambda^{84}+3\lambda^{86}+\lambda^{90}$

$\Phi_{34}(\lambda)(1-\lambda^6)^2(1-\lambda^{12})^2(1-\lambda^{18})^2(1-\lambda^{24})^2(1-\lambda^{30})^2(1-\lambda^{42})^2 = 1+5\lambda^6+26\lambda^{12}+121\lambda^{18}+493\lambda^{24}+1708\lambda^{30}+5171\lambda^{36}+13870\lambda^{42}+33231\lambda^{48}+72049\lambda^{54}+142690\lambda^{60}+260289\lambda^{66}+440521\lambda^{72}+695882\lambda^{78}+1031256\lambda^{84}+1439572\lambda^{90}+1899382\lambda^{96}+2374985\lambda^{102}+2820136\lambda^{108}+3185028\lambda^{114}+3424853\lambda^{120}+3508502\lambda^{126}+3424853\lambda^{132}+3185028\lambda^{138}+2820136\lambda^{144}+2374985\lambda^{150}+1899382\lambda^{156}+1439572\lambda^{162}+1031256\lambda^{168}+695882\lambda^{174}+440521\lambda^{180}+260289\lambda^{186}+142690\lambda^{192}+72049\lambda^{198}+33231\lambda^{204}+13870\lambda^{210}+5171\lambda^{216}+1708\lambda^{222}+493\lambda^{228}+121\lambda^{234}+26\lambda^{240}+5\lambda^{246}+\lambda^{252}$

$\Phi_{35}(\lambda)(1-\lambda^2)^2(1-\lambda^5)^2(1-\lambda^6)^2(1-\lambda^8)^2(1-\lambda^9)^2(1-\lambda^{12})^2 = 1+\lambda^2+\lambda^4+4\lambda^5+6\lambda^6+4\lambda^7+14\lambda^8+16\lambda^9+24\lambda^{10}+34\lambda^{11}+57\lambda^{12}+60\lambda^{13}+105\lambda^{14}+122\lambda^{15}+168\lambda^{16}+224\lambda^{17}+307\lambda^{18}+328\lambda^{19}+473\lambda^{20}+552\lambda^{21}+660\lambda^{22}+780\lambda^{23}+993\lambda^{24}+1036\lambda^{25}+1268\lambda^{26}+1404\lambda^{27}+1564\lambda^{28}+1688\lambda^{29}+1945\lambda^{30}+1928\lambda^{31}+2150\lambda^{32}+2226\lambda^{33}+2284\lambda^{34}+2266\lambda^{35}+2454\lambda^{36}+2266\lambda^{37}+2284\lambda^{38}+2226\lambda^{39}+2150\lambda^{40}+1928\lambda^{41}+1945\lambda^{42}+1688\lambda^{43}+1564\lambda^{44}+1404\lambda^{45}+1268\lambda^{46}+1036\lambda^{47}+993\lambda^{48}+780\lambda^{49}+660\lambda^{50}+552\lambda^{51}+473\lambda^{52}+328\lambda^{53}+307\lambda^{54}+224\lambda^{55}+168\lambda^{56}+122\lambda^{57}+105\lambda^{58}+60\lambda^{59}+57\lambda^{60}+34\lambda^{61}+24\lambda^{62}+16\lambda^{63}+14\lambda^{64}+4\lambda^{65}+6\lambda^{66}+4\lambda^{67}+\lambda^{68}+\lambda^{70}+\lambda^{72}$

$\Phi_{36}(\lambda)(1-\lambda^2)^2(1-\lambda^6)^2(1-\lambda^8)^2(1-\lambda^{10})^2(1-\lambda^{12})^2(1-\lambda^{14})^2(1-\lambda^{18})^2 = 1+\lambda^2+\lambda^4+6\lambda^6+13\lambda^8+25\lambda^{10}+58\lambda^{12}+113\lambda^{14}+207\lambda^{16}+401\lambda^{18}+704\lambda^{20}+1195\lambda^{22}+2021\lambda^{24}+3235\lambda^{26}+5022\lambda^{28}+7648\lambda^{30}+11203\lambda^{32}+15957\lambda^{34}+22254\lambda^{36}+30036\lambda^{38}+39534\lambda^{40}+50917\lambda^{42}+63737\lambda^{44}+77953\lambda^{46}+93312\lambda^{48}+108846\lambda^{50}+124198\lambda^{52}+138774\lambda^{54}+151313\lambda^{56}+161524\lambda^{58}+168880\lambda^{60}+172431\lambda^{62}+172431\lambda^{64}+168880\lambda^{66}+161524\lambda^{68}+151313\lambda^{70}+138774\lambda^{72}+124198\lambda^{74}+108846\lambda^{76}+93312\lambda^{78}+77953\lambda^{80}+63737\lambda^{82}+50917\lambda^{84}+39534\lambda^{86}+30036\lambda^{88}+22254\lambda^{90}+15957\lambda^{92}+11203\lambda^{94}+7648\lambda^{96}+5022\lambda^{98}+3235\lambda^{100}+2021\lambda^{102}+1195\lambda^{104}+704\lambda^{106}+401\lambda^{108}+207\lambda^{110}+113\lambda^{112}+58\lambda^{114}+25\lambda^{116}+13\lambda^{118}+6\lambda^{120}+\lambda^{122}+\lambda^{124}+\lambda^{126}$

$$\Phi_{37}(\lambda)(1-\lambda^2)^2(1-\lambda^8)^2(1-\lambda^{12})^2(1-\lambda^{14})^2(1-\lambda^{18})^2(1-\lambda^{20})^2(1-\lambda^{24})^2(1-\lambda^{30})^2 = 1+\lambda^2+\lambda^4+\lambda^6+8\lambda^8+8\lambda^{10}+20\lambda^{12}+40\lambda^{14}+68\lambda^{16}+114\lambda^{18}+231\lambda^{20}+362\lambda^{22}+627\lambda^{24}+1071\lambda^{26}+1729\lambda^{28}+2742\lambda^{30}+4511\lambda^{32}+6826\lambda^{34}+10559\lambda^{36}+16133\lambda^{38}+23808\lambda^{40}+34877\lambda^{42}+51058\lambda^{44}+72085\lambda^{46}+101941\lambda^{48}+142182\lambda^{50}+194283\lambda^{52}+263558\lambda^{54}+354458\lambda^{56}+466884\lambda^{58}+612511\lambda^{60}+793616\lambda^{62}+1013302\lambda^{64}+1284727\lambda^{66}+1612626\lambda^{68}+1995173\lambda^{70}+2455063\lambda^{72}+2986700\lambda^{74}+3592478\lambda^{76}+4291925\lambda^{78}+5076549\lambda^{80}+5938511\lambda^{82}+6905754\lambda^{84}+7946429\lambda^{86}+9059355\lambda^{88}+10259172\lambda^{90}+11507148\lambda^{92}+12789531\lambda^{94}+14127342\lambda^{96}+15451929\lambda^{98}+16765458\lambda^{100}+18068952\lambda^{102}+19295243\lambda^{104}+20440562\lambda^{106}+21516440\lambda^{108}+22435015\lambda^{110}+23224382\lambda^{112}+23876764\lambda^{114}+24326616\lambda^{116}+24604990\lambda^{118}+24720700\lambda^{120}+24604990\lambda^{122}+24326616\lambda^{124}+23876764\lambda^{126}+23224382\lambda^{128}+22435015\lambda^{130}+21516440\lambda^{132}+20440562\lambda^{134}+19295243\lambda^{136}+18068952\lambda^{138}+16765458\lambda^{140}+15451929\lambda^{142}+14127342\lambda^{144}+12789531\lambda^{146}+11507148\lambda^{148}+10259172\lambda^{150}+9059355\lambda^{152}+7946429\lambda^{154}+6905754\lambda^{156}+5938511\lambda^{158}+5076549\lambda^{160}+4291925\lambda^{162}+3592478\lambda^{164}+2986700\lambda^{166}+2455063\lambda^{168}+1995173\lambda^{170}+1612626\lambda^{172}+1284727\lambda^{174}+1013302\lambda^{176}+793616\lambda^{178}+612511\lambda^{180}+466884\lambda^{182}+354458\lambda^{184}+263558\lambda^{186}+194283\lambda^{188}+142182\lambda^{190}+101941\lambda^{192}+72085\lambda^{194}+51058\lambda^{196}+34877\lambda^{198}+23808\lambda^{200}+16133\lambda^{202}+10559\lambda^{204}+6826\lambda^{206}+4511\lambda^{208}+2742\lambda^{210}+1729\lambda^{212}+1071\lambda^{214}+627\lambda^{216}+362\lambda^{218}+231\lambda^{220}+114\lambda^{222}+68\lambda^{224}+40\lambda^{226}+20\lambda^{228}+8\lambda^{230}+8\lambda^{232}+\lambda^{234}+\lambda^{236}+\lambda^{238}+\lambda^{240}$$

Remark. The remaining cases of irreducible finite unitary reflection groups are investigated in [18].

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References

1. E. Bannai and M. Ozeki, *Construction of Jacobi forms from certain polynomials, preprint.*
2. E. Bannai, S. Minashima and M. Ozeki, *On Jacobi forms of weight 4, preprint.*.
3. C. Chevalley, *Invariants of finite groups generated by reflections*, Amer. J. Math. **77** (1955), 778–782.
4. Arjeh M. Cohen, *Finite complex reflection groups*, Ann. Scie. Ecole. Norm Sup. ser. 4 **,tom. 9** (1976), 379–436.
5. H. S. M. Coxeter, *The product of the generators of a finite group generated by reflections*, Duke Math. J. **18** (1955), 765–782.
6. ______, *Finite groups generated by unitary reflections*, Abhand. Math. **31** (1967), 25–135.
7. J. S. Frame, *The classes and representations of the groups of 27 lines and 28 bitangents*, Analli di Math. **32** (1951), 83–119.
8. ______, *The characters of the Weyl group E_8*, Computational Problems in Abstract Algebra ed.J. Leech, 1970.
9. Larry C. Grove, *The characters of the hecatinicisahedroidal group*, J. für Math. **265** (1972), 160–169.
10. C. M. Hamill, *On a finite group of order 6,531,840*, Proc. London Math. Soc. (2) **52** (1951), 401–454.
11. M. Schönert, et. al., *GAP: Groups, Algorithms and Programming* (1993), Lehrstuhl D für Mathematik, RWTH Aachen.
12. G. C. Shephard, *Unitary groups generated by reflections*, Cand. J. Math., **5** (1953), 364–383.
13. ______, *Abstract definitions for reflection groups*, Cand. J. Math. **9** (1957), 273–276.
14. G. C. Shephard and J. A. Todd, *Finite unitary reflection groups*, Canad. J. Math. **6** (1954), 274–304.
15. N. J. A. Sloane, *Error-correcting codes and invariant theory:New applications of a nineteenth-century technique*, Amer. Math. Monthly **84** (1977), 82–107.
16. Richard P. Stanley, *Invariants of finite groups and their applications to combinatorics*, Bull. Amer. Math. Soc. **1** (1979), 475–511.
17. Bernd Sturmfels, *Algorithms in invariant theory*, Springer-Verlag, 1993.
18. K. Tanabe, *The Molien series for the invariant ring associated with the unitary reflection group $G(m,p,n)$, preprint* (1995).
19. J. A. Todd, *The invariants of a finite collineation group in five dimensions*, Proc. Cambridge Phil. Soc. **46** (1950), 73–90.
20. P. Du. Val, *Homographies, quaternions and rotations*, Clarendon Press, Oxford, 1964.

i	order	c_i	$\det(I-\lambda c_i)$	$\lvert C_i \rvert$
1	1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\lambda^2 - 2\lambda + 1$	1
2	2	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$	$\lambda^2 + 2\lambda + 1$	1
3	2	$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$	$-\lambda^2 + 1$	6
4	2	$\frac{1}{2}\begin{bmatrix} -\zeta_8 + \zeta_8^3 & \zeta_8 + \zeta_8^3 \\ -\zeta_8 - \zeta_8^3 & \zeta_8 - \zeta_8^3 \end{bmatrix}$	$-\lambda^2 + 1$	12
5	3	$\frac{1}{2}\begin{bmatrix} -1 + \zeta_4 & -1 + \zeta_4 \\ 1 + \zeta_4 & -1 - \zeta_4 \end{bmatrix}$	$\lambda^2 + \lambda + 1$	8
6	4	$\begin{bmatrix} \zeta_4 & 0 \\ 0 & \zeta_4 \end{bmatrix}$	$-\lambda^2 - 2\zeta_4\lambda + 1$	1
7	4	$\begin{bmatrix} -\zeta_4 & 0 \\ 0 & -\zeta_4 \end{bmatrix}$	$-\lambda^2 + 2\zeta_4\lambda + 1$	1
8	4	$\begin{bmatrix} 0 & -\zeta_4 \\ -\zeta_4 & 0 \end{bmatrix}$	$\lambda^2 + 1$	6
9	4	$\begin{bmatrix} -1 & 0 \\ 0 & -\zeta_4 \end{bmatrix}$	$\zeta_4\lambda^2 + (1+\zeta_4)\lambda + 1$	6
10	4	$\begin{bmatrix} \zeta_4 & 0 \\ 0 & 1 \end{bmatrix}$	$\zeta_4\lambda^2 - (1+\zeta_4)\lambda + 1$	6
11	4	$\begin{bmatrix} -\zeta_4 & 0 \\ 0 & 1 \end{bmatrix}$	$-\zeta_4\lambda^2 + (-1+\zeta_4)\lambda + 1$	6
12	4	$\frac{1}{2}\begin{bmatrix} -1 + \zeta_4 & -1 + \zeta_4 \\ 1 - \zeta_4 & -1 + \zeta_4 \end{bmatrix}$	$-\zeta_4\lambda^2 + (1-\zeta_4)\lambda + 1$	6
13	4	$\frac{1}{2}\begin{bmatrix} \zeta_8 + \zeta_8^3 & -\zeta_8 + \zeta_8^3 \\ \zeta_8 - \zeta_8^3 & -\zeta_8 - \zeta_8^3 \end{bmatrix}$	$\lambda^2 + 1$	12
14	6	$\frac{1}{2}\begin{bmatrix} 1 + \zeta_4 & -1 - \zeta_4 \\ 1 - \zeta_4 & 1 - \zeta_4 \end{bmatrix}$	$\lambda^2 - \lambda + 1$	8
15	8	$\begin{bmatrix} \zeta_8 & 0 \\ 0 & \zeta_8 \end{bmatrix}$	$\zeta_4\lambda^2 - 2\zeta_8\lambda + 1$	1

Appendix 1.

We give information on conjugacy classes of G, where G is the group of $No.9$. Order denotes the order of c_i.

i	order	c_i	$\det(I-\lambda c_i)$	$\|C_i\|$
16	8	$\begin{bmatrix} -\zeta_8 & 0 \\ 0 & -\zeta_8 \end{bmatrix}$	$\zeta_4\lambda^2+2\zeta_8\lambda+1$	1
17	8	$\begin{bmatrix} \zeta_8^3 & 0 \\ 0 & \zeta_8^3 \end{bmatrix}$	$-\zeta_4\lambda^2-2\zeta_8^3\lambda+1$	1
18	8	$\begin{bmatrix} -\zeta_8^3 & 0 \\ 0 & -\zeta_8^3 \end{bmatrix}$	$-\zeta_4\lambda^2+2\zeta_8^3\lambda+1$	1
19	8	$\begin{bmatrix} \zeta_8^3 & 0 \\ 0 & -\zeta_8^3 \end{bmatrix}$	$\zeta_4\lambda^2+1$	6
20	8	$\begin{bmatrix} 0 & \zeta_8^3 \\ -\zeta_8^3 & 0 \end{bmatrix}$	$-\zeta_4\lambda^2+1$	6
21	8	$\frac{1}{2}\begin{bmatrix} \zeta_8-\zeta_8^3 & -\zeta_8-\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & \zeta_8-\zeta_8^3 \end{bmatrix}$	$\lambda^2+(-\zeta_8+\zeta_8^3)\lambda+1$	6
22	8	$\frac{1}{2}\begin{bmatrix} -\zeta_8+\zeta_8^3 & -\zeta_8-\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & -\zeta_8+\zeta_8^3 \end{bmatrix}$	$\lambda^2+(\zeta_8-\zeta_8^3)\lambda+1$	6
23	8	$\begin{bmatrix} -\zeta_8 & 0 \\ 0 & -\zeta_8^3 \end{bmatrix}$	$-\lambda^2+(\zeta_8+\zeta_8^3)\lambda+1$	6
24	8	$\frac{1}{2}\begin{bmatrix} \zeta_8+\zeta_8^3 & \zeta_8+\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & \zeta_8+\zeta_8^3 \end{bmatrix}$	$-\lambda^2-(\zeta_8+\zeta_8^3)\lambda+1$	6
25	8	$\begin{bmatrix} 0 & -1 \\ -\zeta_4 & 0 \end{bmatrix}$	$-\zeta_4\lambda^2+1$	12
26	8	$\frac{1}{2}\begin{bmatrix} -1+\zeta_4 & 1-\zeta_4 \\ 1-\zeta_4 & 1-\zeta_4 \end{bmatrix}$	$\zeta_4\lambda^2+1$	12
27	12	$\frac{1}{2}\begin{bmatrix} -1-\zeta_4 & 1+\zeta_4 \\ 1-\zeta_4 & 1-\zeta_4 \end{bmatrix}$	$-\lambda^2+\zeta_4\lambda+1$	8
28	12	$\frac{1}{2}\begin{bmatrix} -1+\zeta_4 & -1+\zeta_4 \\ -1-\zeta_4 & 1+\zeta_4 \end{bmatrix}$	$-\lambda^2-\zeta_4\lambda+1$	8
29	24	$\frac{1}{2}\begin{bmatrix} \zeta_8+\zeta_8^3 & \zeta_8-\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & \zeta_8-\zeta_8^3 \end{bmatrix}$	$\zeta_4\lambda^2-\zeta_8\lambda+1$	8
30	24	$\frac{1}{2}\begin{bmatrix} \zeta_8+\zeta_8^3 & -\zeta_8+\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & -\zeta_8+\zeta_8^3 \end{bmatrix}$	$-\zeta_4\lambda^2-\zeta_8^3\lambda+1$	8
31	24	$\frac{1}{2}\begin{bmatrix} -\zeta_8+\zeta_8^3 & -\zeta_8-\zeta_8^3 \\ \zeta_8-\zeta_8^3 & -\zeta_8-\zeta_8^3 \end{bmatrix}$	$\zeta_4\lambda^2+\zeta_8\lambda+1$	8
32	24	$\frac{1}{2}\begin{bmatrix} -\zeta_8-\zeta_8^3 & -\zeta_8+\zeta_8^3 \\ -\zeta_8-\zeta_8^3 & \zeta_8-\zeta_8^3 \end{bmatrix}$	$-\zeta_4\lambda^2+\zeta_8^3\lambda+1$	8

Appendix 1 (continued).

α	$\|D_\alpha\|$	d_α
1	1	$(1-\lambda)^6$
2	126	$(1-\lambda)^4(1-\lambda^2)$
3	2835	$(1-\lambda)^2(1-\lambda^2)^2$
4	3360	$(1-\lambda)^3(1-\lambda^3)$
5	5670	$(1-\lambda^2)^3$
6	60480	$(1-\lambda)(1-\lambda^2)(1-\lambda^3)$
7	60480	$(1-\lambda)^2(1-\lambda^4)$
8	560	$(1-\lambda)^3(1-\zeta_3\lambda)^3$
$9,1$	5040	$(1-\lambda)^3(\zeta_3-\lambda)(\zeta_3^2-\lambda^2)$
$9,2$	5040	$(1-\lambda)^3(\zeta_3^2-\lambda)(\zeta_3-\lambda^2)$
10	90720	$(1-\lambda^2)^2(1+\lambda+\lambda^2)$
11	40320	$(1-\lambda^3)^2$
12	204120	$(1-\lambda^2)(1-\lambda^4)$
13	45360	$(1-\lambda)(\zeta_3-\lambda)(1-\lambda^2)(\zeta_3^2-\lambda^2)$
14	653184	$(1-\lambda)(1-\lambda^5)$
15	34020	$(1-\lambda)^2(1+\lambda^2)^2$
16	3360	$(1-\lambda^3)^2$
17	90720	$(1-\lambda)^2(1+\lambda^2+\lambda^4)$
$18,1$	120960	$(1-\lambda)^2(\zeta_3-\lambda)(\zeta_3^2-\lambda^3)$
$18,2$	120960	$(1-\lambda)^2(\zeta_3^2-\lambda)(\zeta_3-\lambda^3)$
19	30240	$(1-\lambda)^2(\zeta_3-\lambda)^2(\zeta_3-\lambda^2)$
20	544320	$(1-\lambda^4)(1+\lambda+\lambda^2)$
21	653184	$(1-\lambda^5)(1+\lambda)$
22	362880	$1-\lambda^6$
23	90720	$1-\lambda^6$

Appendix 2

G is the group $No.34$ in [14]. The numbering follows [10], [19].

α	$\lvert D_\alpha \rvert$	d_α
$24,1$	362880	$(\zeta_3-\lambda)(1-\lambda^2)(\zeta_3^2-\lambda^3)$
$24,2$	362880	$(\zeta_3^2-\lambda)(1-\lambda^2)(\zeta_3-\lambda^3)$
25	408240	$(1-\lambda^2)(1+\lambda^4)$
26	34020	$(1-\lambda^4)(1+\lambda^2)$
27	272160	$(1-\lambda)(1+\zeta_3^2\lambda)(1-\zeta_3^2\lambda^2+\zeta_3\lambda^4)$
28	272160	$(1+\lambda)(1+\zeta_3^2\lambda)(1-\zeta_3^2\lambda^2+\zeta_3\lambda^4)$
29	241920	$1+\lambda^3+\lambda^6$
30	933120	$1+\lambda+\lambda^2+\lambda^3+\lambda^4+\lambda^5+\lambda^6$
31	408240	$(1+\lambda^2)(1+\lambda^4)$

Appendix 2 (contineud).

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